Written exam at the Department of Economics
Summer 2020 – Reexam
Monetary Policy
Suggested Answers

PROBLEM A

1) False. When both prices and wages are sticky, it will in general not be possible for the central bank to obtain a gapless allocation where the output gap, price and wage inflation are zero. This is because technology shocks lead to changes in the natural real wage. The optimal monetary policy therefore strikes a balance between limiting price and wage inflation (as well as output gap fluctuations) and allowing the real wage to track its natural counterpart. This implies that the rate of wage inflation will not be constant at zero.

2) False. Unconventional monetary policies include both quantitative easing, credit easing, and forward guidance. Only the first of these involves an expansion of the central bank's balance sheet. Thus, the statement is false. In practice, many central banks have adopted a mix of these policies, and have thus seen their balance sheets expand, but this does not necessarily have to be the case.

3) False. It is true that inflation and the output gap are positively correlated: A positive monetary policy shock for example puts downward pressure on real rates, thus output via the IS curve, and hence inflation via firm's pricing decisions. The nominal interest rate however may either rise or fall: If the monetary policy shock is sufficiently persistent, it falls in order to offset the strong recession and deflation induced by the initial positive monetary policy shock.

PROBLEM B

1) Equation B.1 is an open-economy version of the New-Keynesian Phillips curve (NKPC). For given inflation expectations, it implies a positive relationship between domestic producer price inflation and the domestic output gap: An increase in the output gap raises the marginal cost faced by firms. This leads to

higher prices and hence inflation. Moreover, the NKPC also implies that current inflation increases if agents expect inflation to increase in the future: Since firms are subject to sticky prices, they know that they may not be able to change their price for some periods into the future. Hence, if they expect high inflation, it will be optimal for them to raise their price already today, if allowed to. The NKPC is derived from the pricing decisions of firms, taking into account the labor supply decision of households as well as the production function, which both affect the marginal cost faced by firms. Importantly, this open-economy version of the NKPC features domestic producer price inflation (as opposed to consumer price inflation), since it is derived from firms' price-setting problem. It may also be noted that the slope of the open-economy NKPC is given by κ_ν, which may be larger or smaller than (or equal to) the corresponding slope parameter κ in a closed economy.

Equation B.2 is the dynamic IS curve (DIS). It is derived by combining the household's Euler equation for consumption with the goods market clearing condition. It implies a relationship between the current and future domestic output gap and the expected real interest rate (in deviations from its natural or steady state level): When the real interest rate is (expected to be) high, saving for the future becomes more attractive, and current consumption therefore less attractive. As a result, economic activity is moved from the present to the future, so the current output gap drops (becomes negative), while the future output gap increases. Also, the DIS implies that an increase in the expected future output gap will drive up the current output gap due to households' desire to smooth consumption over time. Here the DIS is written in terms of domestic producer price inflation, although it would be possible to rewrite it in terms of domestic consumer price inflation instead.

Equation B.3 is an interest rate rule specifying how the central bank sets the interest rate in response to movements in inflation, the output gap, and the natural output gap. It is often referred to as a *Taylor rule*. The assumed parameter values imply that the central bank raises the nominal interest rate when inflation is high (i.e., above its target) and/or when the output gap is positive. In particular, the assumption that $\phi_\pi > 1$ ensures that the central bank raises the nominal interest rate more than one-for-one in response to an increase in inflation, thereby raising also the real interest rate, as follows from the Fisher equation ($r_t = i_t - E_t\pi_{t+1}$). In turn, this depresses current economic activity, and thereby brings inflation back down. This assumption, which is often referred to as the *Taylor principle*, is necessary to ensure a unique determinate equilibrium.

Equation B.4 is the uncovered interest rate parity (UIP) condition. It is an arbitrage condition: For an investor to be indifferent between investing in the domestic or the foreign economy, it must hold. If the investor can earn a higher nominal interest rate in the domestic economy, it must be because she is expecting to incur a loss from the expected movements in the nominal exchange rate, i.e., that she is expecting the currency of the domestic country to depreciate (increase) against the foreign currency.

2) Note first that we can write the natural interest rate as:

$$\begin{aligned} r_t^n &= \rho + \sigma_\nu E_t \Delta y_{t+1}^n \Leftrightarrow \\ r_t^n &= \rho + \sigma_\nu E_t \left(y_{t+1}^n - y_t^n \right) \Leftrightarrow \\ r_t^n &= \rho + \sigma_\nu E_t \left(\Gamma_a a_{t+1} - \Gamma_a a_t \right) \Leftrightarrow \\ r_t^n &= \rho - \sigma_\nu \Gamma_a a_t, \end{aligned}$$

where we have used that the expectation of the shock tomorrow is simply zero, as indicated in the exercise text. Further, since $y^n = 0$, we also have:

$$\widehat{y}_t^n \equiv y_t^n - y^n = \Gamma_a a_t.$$

Now conjecture that:

$$\begin{aligned} \pi_{H,t} &= c_1 a_t, \\ \tilde{y}_t &= c_2 a_t. \end{aligned}$$

Note first that this has the following implications for the expectations of these two variables:

$$\begin{aligned} \mathrm{E}_t \pi_{H,t+1} &= c_1 \mathrm{E}_t a_{t+1} \Leftrightarrow \\ \mathrm{E}_t \pi_{H,t+1} &= 0, \end{aligned}$$

$$\mathrm{E}_t \tilde{y}_{t+1} = 0.$$

Now insert the guesses and these expectations into the original system of equations. First into the NKPC:

$$\begin{aligned}
\pi_{H,t} &= \beta \mathrm{E}_t \pi_{H,t+1} + \kappa_\nu \tilde{y}_t \Leftrightarrow \\
c_1 a_t &= 0 + \kappa_\nu c_2 a_t \Leftrightarrow \\
c_1 &= \kappa_\nu c_2 .
\end{aligned}$$

Now operate on the DIS, first using the Taylor rule, the natural interest rate expression, and that for the natural output gap:

$$\begin{aligned}
\tilde{y}_t &= \mathrm{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma_\nu} \left(i_t - \mathrm{E}_t \pi_{H,t+1} - r_t^n \right) \Leftrightarrow \\
\tilde{y}_t &= \mathrm{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma_\nu} \left(\left[\rho + \phi_\pi \pi_{H,t} + \phi_y \widetilde{y}_t + \phi_y \widehat{y}_t^n \right] - \mathrm{E}_t \pi_{H,t+1} - \left[\rho - \sigma_\nu \Gamma_a a_t \right] \right) \Leftrightarrow \\
\tilde{y}_t &= \mathrm{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma_\nu} \left(\phi_\pi \pi_{H,t} + \phi_y \widetilde{y}_t + \phi_y \Gamma_a a_t - \mathrm{E}_t \pi_{H,t+1} + \sigma_\nu \Gamma_a a_t \right) .
\end{aligned}$$

Insert from the conjecture:

$$\begin{aligned}
\tilde{y}_t &= \mathrm{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma_\nu} \left(\phi_\pi \pi_{H,t} + \phi_y \widetilde{y}_t + \phi_y \Gamma_a a_t - \mathrm{E}_t \pi_{H,t+1} + \sigma_\nu \Gamma_a a_t \right) \Leftrightarrow \\
c_2 a_t &= 0 - \frac{1}{\sigma_\nu} \left(\phi_\pi c_1 a_t + \phi_y c_2 a_t + \phi_y \Gamma_a a_t - 0 + \sigma_\nu \Gamma_a a_t \right) \Leftrightarrow \\
c_2 &= -\frac{1}{\sigma_\nu} \left(\phi_\pi c_1 + \phi_y c_2 + \phi_y \Gamma_a + \sigma_\nu \Gamma_a \right) \Leftrightarrow \\
c_2 \left[1 + \frac{\phi_y}{\sigma_\nu} \right] &= -\frac{\phi_\pi}{\sigma_\nu} c_1 - \frac{\Gamma_a}{\sigma_\nu} \left(\phi_y + \sigma_\nu \right) \Leftrightarrow \\
c_2 &= -\frac{\phi_\pi}{\sigma_\nu + \phi_y} c_1 - \frac{\Gamma_a}{\sigma_\nu + \phi_y} \left(\phi_y + \sigma_\nu \right) .
\end{aligned}$$

Now insert into the expression derived from the NKPC:

$$\begin{aligned}
c_1 &= \kappa_\nu c_2 \Leftrightarrow \\
c_1 &= -\frac{\phi_\pi \kappa_\nu}{\sigma_\nu + \phi_y} c_1 - \frac{\Gamma_a \kappa_\nu}{\sigma_\nu + \phi_y} \left(\phi_y + \sigma_\nu \right) \Leftrightarrow \\
c_1 \left[1 + \frac{\phi_\pi \kappa_\nu}{\sigma_\nu + \phi_y} \right] &= -\frac{\Gamma_a \kappa_\nu}{\sigma_\nu + \phi_y} \left(\phi_y + \sigma_\nu \right) \Leftrightarrow \\
c_1 &= -\frac{\Gamma_a \kappa_\nu \left(\phi_y + \sigma_\nu \right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} .
\end{aligned}$$

Then plug back in to solve for c_2:

$$\begin{aligned} c_1 &= \kappa_\nu c_2 \Leftrightarrow \\ c_2 &= -\frac{\Gamma_a \left(\phi_y + \sigma_\nu\right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu}. \end{aligned}$$

So we have the following solutions:

$$\pi_{H,t} = -\frac{\Gamma_a \kappa_\nu \left(\phi_y + \sigma_\nu\right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} a_t, \tag{1}$$

$$\tilde{y}_t = -\frac{\Gamma_a \left(\phi_y + \sigma_\nu\right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} a_t. \tag{2}$$

This shows that in response to a positive technology shock, both domestic inflation and the output gap decline. The reason is the following: The shock reduces the marginal cost of firms, who respond by reducing their prices when able to do so. This leads to a drop in inflation. At the same time, while output increases, it falls short of the increase in natural output, because some prices cannot be changed and therefore remain too high, thus reducing actual demand. This means that the output gap drops. These responses are qualitatively in line with those observed in a closed economy.

3) Insert the solutions for $\widetilde{y}_t$, $\widehat{y}_t^n$, and π_t in the interest rate rule:

$$\begin{aligned} i_t &= \rho + \phi_\pi \pi_{H,t} + \phi_y \widetilde{y}_t + \phi_y \widehat{y}_t^n \Leftrightarrow \\ i_t &= \rho - \phi_\pi \frac{\Gamma_a \kappa_\nu \left(\phi_y + \sigma_\nu\right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} a_t - \phi_y \frac{\Gamma_a \left(\phi_y + \sigma_\nu\right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} a_t + \phi_y \Gamma_a a_t \Leftrightarrow \\ i_t &= \rho - \Gamma_a \left[\frac{\phi_\pi \kappa_\nu \left(\phi_y + \sigma_\nu\right) + \phi_y \left(\phi_y + \sigma_\nu\right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} - \phi_y \right] a_t \Leftrightarrow \\ i_t &= \rho - \Gamma_a \left[\frac{\phi_\pi \kappa_\nu \left(\phi_y + \sigma_\nu\right) + \phi_y \left(\phi_y + \sigma_\nu\right) - \phi_y \left(\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu\right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} \right] a_t \Leftrightarrow \\ i_t &= \rho - \frac{\Gamma_a \phi_\pi \kappa_\nu \sigma_\nu}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} a_t, \end{aligned}$$

from which it follows that the central bank reduces the nominal interest rate in response to a positive technology shock, so as to accommodate the shock. This is in line with a closed-economy model. If we combine this with the uncovered interest parity (UIP), we obtain:

$$\begin{aligned} i_t &= i_t^* + \mathrm{E}_t \{e_{t+1}\} - e_t \Leftrightarrow \\ \rho - \frac{\Gamma_a \phi_\pi \kappa_\nu \sigma_\nu}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} a_t &= \rho + \mathrm{E}_t \{e_{t+1}\} - e_t \Leftrightarrow \\ e_t &= \frac{\Gamma_a \phi_\pi \kappa_\nu \sigma_\nu}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} a_t + \mathrm{E}_t \{e_{t+1}\}. \end{aligned}$$

If we follow the hint and take $\mathrm{E}_t \{e_{t+1}\}$ as given, it is easy to see that the nominal exchange rate needs to increase/depreciate. More generally (i.e., without taking $\mathrm{E}_t \{e_{t+1}\}$ as given), we can see that $\mathrm{E}_t \{\Delta e_{t+1}\} \equiv \mathrm{E}_t \{e_{t+1}\} - e_t$ needs to drop, which implies that the nominal exchange rate must be expected to appreciate in the future, which requires that it depreciates or increases today, so the same intuition applies (students get extra credit if they make this argument). The reason is that according to the UIP, investors must be expecting to gain from a future exchange rate appreciation if they are to be willing to hold their money in the domestic economy and earn a lower interest rate. From the viewpoint of domestic consumers, an exchange rate depreciation today means that the domestic currency is weaker, which makes it more expensive for them to consume foreign goods.

4) We begin by using the hint that $e_0 = 0$ and thus $\Delta e_t = e_t$ when considering the impact effect of the shock. We can thus write the expression for consumer price inflation as:

$$\begin{aligned} \pi_t &= (1-\nu)\,\pi_{H,t} + \nu \Delta e_t \Leftrightarrow \\ \pi_t &= (1-\nu)\,\pi_{H,t} + \nu e_t \Leftrightarrow \\ \pi_t &= -(1-\nu)\frac{\Gamma_a \kappa_\nu \left(\phi_y + \sigma_\nu\right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} a_t + \nu \left[\frac{\Gamma_a \phi_\pi \kappa_\nu \sigma_\nu}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} a_t + \mathrm{E}_t \{e_{t+1}\}\right], \end{aligned}$$

from which it is easy to obtain (maintaining our - somewhat inaccurate - assumption that $\mathrm{E}_t \{e_{t+1}\}$ does not respond to a_t):

$$\frac{\partial \pi_t}{\partial a_t} = -\frac{(1-\nu)\,\Gamma_a \kappa_\nu \left(\phi_y + \sigma_\nu\right)}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu} + \frac{\nu \Gamma_a \phi_\pi \kappa_\nu \sigma_\nu}{\sigma_\nu + \phi_y + \phi_\pi \kappa_\nu}.$$

From this expression, we see that it is not possible to determine the effect on domestic consumer price inflation of a domestic technology shock, as we cannot say whether the positive or the negative term dominates. The reason is that two opposing forces are at play: On one hand, the technology shock reduces the domestic producer price inflation ($\pi_{H,t}$), as already shown in Question 2. All else equal, this tends to also reduce consumer price inflation. On the other hand, however, the nominal exchange rate depreciates because of the response of the domestic nominal interest rate. As seen in Question 3, this makes imported goods more expensive for domestic households. It cannot be determined which of these effects dominate. The degree of openness plays a key role in this regard: an increase in the degree of openness (ν) makes it more likely to observe an increase in consumer price inflation, as it increases the share of imported goods AND reduces the share of domestic goods in the consumption basket (this explains why ν appears twice in the partial derivative). Note that students may be able to provide this intuition even without doing any calculations in this question, as they may note it directly from the expression for π_t in the exam paper. This will give partial credit.

PROBLEM C

1) Under discretion, the central bank solves a sequence of static problems:

$$\min_{x_t, \pi_t} \frac{1}{2} \left(\vartheta x_t^2 + \pi_t^2 \right)$$

subject to

$$\pi_t = \kappa x_t + z_t,$$

where $z_t \equiv \beta \mathrm{E}_t \pi_{t+1} + u_t$ is taken as given. We can insert the constraint directly into the minimization problem to obtain:

$$\min_{x_t} \frac{1}{2} \left(\vartheta x_t^2 + (\kappa x_t + z_t)^2 \right).$$

The first-order condition with respect to x_t is thus given by:

$$\begin{aligned} 2\vartheta x_t + 2\kappa (\kappa x_t + z_t) &= 0 \Leftrightarrow \\ \vartheta x_t + \kappa \pi_t &= 0 \Leftrightarrow \\ x_t &= -\frac{\kappa}{\vartheta} \pi_t. \end{aligned}$$

This expression shows that if the rate of inflation increases, the optimal discretionary policy involves a negative output gap, or "leaning against the wind". By pushing the output gap below zero, the central bank can dampen the inflationary pressure in the economy. More generally, the optimality condition reflects the fact that in the face of cost-push shocks to the economy, the central bank faces a trade-off between stabilizing inflation and stabilizing the output gap.

2) Under commitment the central bank no longer takes expectations as given. We set up the problem as a Lagrangian:

$$\mathcal{L} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\vartheta x_t^2 + \pi_t^2 \right) + \mu_t \left(\pi_t - \beta \mathrm{E}_t \pi_{t+1} - \kappa x_t - u_t \right) \right].$$

The first-order conditions with respect to x_t and π_t are now given by:

$$\vartheta x_t - \kappa \mu_t = 0, \qquad t \geq 0,$$

$$\begin{aligned} \pi_t + \mu_t &= 0, \qquad t = 0, \\ \pi_t + \mu_t - \mu_{t-1} &= 0, \qquad t \geq 1. \end{aligned}$$

Combining the first and second equation yields:

$$x_0 = -\frac{\kappa}{\vartheta} \pi_0,$$

while we can insert the first equation into the third to obtain:

$$x_t - x_{t-1} = -\frac{\kappa}{\vartheta} \pi_t, \qquad t \geq 1.$$

3) In comparison to the optimal policy under discretion, the optimal commitment policy features history dependence: A recession yesterday translates into a continued recession today. For a given path of above-trend inflation $\{\pi_t\}_{t=0,1,\ldots}$, for example, the FOCs imply that the recession x_0 that is generated due to π_0 facilitates a milder recession x_1 than under discretion (for given inflation paths). In other words, the policy trade-off is improved. To see how this works intuitively, consider a cost-push shock that drives up inflation. As discussed in the case of discretionary policy, this implies that the central bank needs to run contractionary monetary policy to generate a negative output gap, i.e. a recession. Under commitment, the central bank can credibly commit to a contractionary policy also in the next period. This will bring down inflation (as well as the output gap) in the next period, and therefore bring down current inflation expectations. In turn, this dampens the current inflation rate, since firms are forward-looking when making their price-setting decisions. In other words, the ability to affect

inflation expectations means that the central bank does not need to bring down the current output gap by as much. It is optimal for the central bank to create a mild but prolonged recession since the loss function is quadratic in the deviations of inflation and output gap from their targets.